# Série N°1

✍ Continuité

✍ Dérivation

✍ Étude des fonctions

## Exercice 1

Calculer les limites suivantes :

- $\lim\limits_{x\to+\infty} \frac{\sqrt{2}-2x^2}{x+1}$
- $\lim\limits_{x\to-\infty} \frac{x^3+1}{2x^4-4}$
- $\lim\limits_{x\to+\infty} \frac{3x^2+1}{x^2+5}$
- $\lim\limits_{x\to 2} \frac{\sqrt{x+2}-2}{x-2}$
- $\lim\limits_{x\to 2} 3x^2-2x+7$
- $\lim\limits_{x\to 1} \sqrt{x^3+x}-x$
- $\lim\limits_{x\to-\infty} -2x^3+2x+4$
- $\lim\limits_{x\to+\infty} 3x^3-2x$
- $\lim\limits_{x\to 3^+} \frac{|3-x|}{x^2-9}$
- $\lim\limits_{x\to 2^+} \frac{2x^2+x}{x^2+4}$
- $\lim\limits_{x\to 1^+} \frac{\sqrt{x^2-x}}{x-1}$
- $\lim\limits_{x\to 0^+} \frac{x-x^2}{\sqrt{x}}$
- $\lim\limits_{x\to-\infty} \sqrt{x^2+1}+x$
- $\lim\limits_{x\to-\infty} \frac{\sqrt{x^2+2x}-1}{x+1}$
- $\lim\limits_{x\to 3} \frac{\sqrt{x}-\sqrt{3}}{\sqrt{x+6}-3}$
- $\lim\limits_{x\to 0} \frac{\sin x-\tan x}{x^3}$
- $\lim\limits_{x\to 4} \frac{x^2-5x+4}{x-4}$
- $\lim\limits_{x\to 0} \frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{x}$
- $\lim\limits_{x\to 0^-} \frac{3x}{x^2+2|x|}$
- $\lim\limits_{x\to 0} \frac{2\sin x-\sin 2x}{x^2}$

## Correction

- $\lim\limits_{x\to+\infty} \frac{\sqrt{2}-2x^2}{x+1} = \lim\limits_{x\to+\infty} \frac{-2x^2}{x} = \lim\limits_{x\to+\infty} -2x = -\infty$
- $\lim\limits_{x\to-\infty} \frac{x^3+1}{2x^4-4} = \lim\limits_{x\to-\infty} \frac{x^3}{2x^4} = \lim\limits_{x\to-\infty} \frac{1}{2x} = 0$
- $\lim\limits_{x\to+\infty} \frac{3x^2+1}{x^2+5} = \lim\limits_{x\to+\infty} \frac{3x^2}{x^2} = 3$
- $\lim\limits_{x\to 2} \frac{\sqrt{x+2}-2}{x-2} = \lim\limits_{x\to 2} \frac{(\sqrt{x+2}-2)(\sqrt{x+2}+2)}{(x-2)(\sqrt{x+2}+2)} = \lim\limits_{x\to 2} \frac{\left(\sqrt{x+2}\right)^2-(2)^2}{(x-2)(\sqrt{x+2}+2)} = \lim\limits_{x\to 2} \frac{x-2}{(x-2)(\sqrt{x+2}+2)} = \lim\limits_{x\to 2} \frac{1}{\sqrt{x+2}+2} = \frac{1}{4}$
- $\lim\limits_{x\to 2} 3x^2-2x+7 = 3\times(2)^2-2\times 2+7 = 15$
- $\lim\limits_{x\to 1} \sqrt{x^3+x}-x = \sqrt{1^3+1}-1 = \sqrt{2}-1$
- $\lim\limits_{x\to-\infty} -2x^3+2x = \lim\limits_{x\to-\infty} -2x^3 = +\infty$
- $\lim\limits_{x\to+\infty} 3x^3-2x = \lim\limits_{x\to+\infty} 3x^3 = +\infty$
- $\lim\limits_{x\to 3^+} \frac{|3-x|}{x^2-9} = \lim\limits_{x\to 3^+} \frac{-(3-x)}{x^2-9} = \lim\limits_{x\to 3^+} \frac{x-3}{(x-3)(x+3)} = \lim\limits_{x\to 3^+} \frac{1}{x+3} = \frac{1}{6}$, car $(x>3 \Rightarrow 3-x<0)$
- $\lim\limits_{x\to 2^+} \frac{2x^2+x}{x^2+4} = \frac{2\times 2^2+2}{2^2+4} = \frac{5}{4}$
- $\lim\limits_{x\to 1^+} \frac{\sqrt{x^2-x}}{x-1} = \lim\limits_{x\to 1^+} \frac{\sqrt{(x-1)(x+1)}}{x-1} = \lim\limits_{x\to 1^+} \frac{\sqrt{(x-1)(x+1)}}{\sqrt{(x-1)^2}} = \lim\limits_{x\to 1^+} \sqrt{\frac{(x-1)(x+1)}{(x-1)^2}} = \lim\limits_{x\to 1^+} \sqrt{\frac{(x+1)}{(x-1)}} = +\infty$

  car : $\lim\limits_{x\to 1^+} x+1 = 2$, $\lim\limits_{x\to 1^+} x-1 = 0^+$ et $x>1$
- $\lim\limits_{x\to 0^+} \frac{x-x^2}{\sqrt{x}} = \lim\limits_{x\to 0^+} \frac{(x-x^2)\times\sqrt{x}}{\sqrt{x}\times\sqrt{x}} = \lim\limits_{x\to 0^+} \frac{x(1-x)\times\sqrt{x}}{x} = \lim\limits_{x\to 0^+} (1-x)\times\sqrt{x} = 0$, car $(x>0)$
- $\lim\limits_{x\to-\infty} \sqrt{x^2+1}+x = \lim\limits_{x\to-\infty} \frac{(\sqrt{x^2+1}+x)(\sqrt{x^2+1}-x)}{\sqrt{x^2+1}-x} = \lim\limits_{x\to-\infty} \frac{1}{\sqrt{x^2+1}-x} = 0$,

  car $(\lim\limits_{x\to-\infty} \sqrt{x^2+1}-x = +\infty)$

- $\lim\limits_{x\to-\infty} \frac{\sqrt{x^2+2x}-1}{x+1} = \lim\limits_{x\to-\infty} \frac{\sqrt{x^2\left(1+\frac{2}{x}\right)}-1}{x+1} = \lim\limits_{x\to-\infty} \frac{|x|\sqrt{\left(1+\frac{2}{x}\right)}-1}{x+1} = \lim\limits_{x\to-\infty} \frac{-x\sqrt{\left(1+\frac{2}{x}\right)}-1}{x+1}$
  $= \lim\limits_{x\to-\infty} \frac{x\left(-\sqrt{\left(1+\frac{2}{x}\right)}-\frac{1}{x}\right)}{x\left(1+\frac{1}{x}\right)} = \lim\limits_{x\to-\infty} \frac{-\sqrt{\left(1+\frac{2}{x}\right)}-\frac{1}{x}}{1+\frac{1}{x}} = -1,$
  car ( $\lim\limits_{x\to-\infty} \frac{2}{x} = 0$ et $\lim\limits_{x\to-\infty} \frac{1}{x} = 0$ )

- $\lim\limits_{x\to3} \frac{\sqrt{x}-\sqrt{3}}{\sqrt{x+6}-3} = \lim\limits_{x\to3} \frac{(\sqrt{x}-\sqrt{3})(\sqrt{x}+\sqrt{3})(\sqrt{x+6}+3)}{(\sqrt{x+6}-3)(\sqrt{x+6}+3)(\sqrt{x}+\sqrt{3})} = \lim\limits_{x\to3} \frac{(x-3)(\sqrt{x+6}+3)}{(x-3)(\sqrt{x}+\sqrt{3})} == \lim\limits_{x\to3} \frac{\sqrt{x+6}+3}{\sqrt{x}+\sqrt{3}} = \frac{6}{2\sqrt{3}} = \sqrt{3}$

- $\lim\limits_{x\to0} \frac{\sin x-\tan x}{x^3} = \lim\limits_{x\to0} \frac{\sin x-\frac{\sin x}{\cos x}}{x^3} = \lim\limits_{x\to0} \frac{\sin x\cos x-\sin x}{\cos x\times x^3} = \lim\limits_{x\to0} \frac{\sin x(\cos x-1)}{\cos x\times x^3} = \lim\limits_{x\to0} \frac{\tan x}{x} \times \frac{-(1-\cos x)}{x^2}$
  $= 1 \times -\frac{1}{2} = -\frac{1}{2}$, car $(\lim\limits_{x\to0} \frac{\tan x}{x} = 1$ et $\lim\limits_{x\to0} \frac{1-\cos x}{x^2} = \frac{1}{2})$

- $\lim\limits_{x\to4} \frac{x^2-5x+4}{x-4} = \lim\limits_{x\to4} \frac{(x-4)(x-1)}{x-4} = \lim\limits_{x\to4} x-1 = 3$

- $\lim\limits_{x\to0} \frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{x} = \lim\limits_{x\to0} \frac{\left(\sqrt{1+x^2}-\sqrt{1-x^2}\right)\left(\sqrt{1+x^2}+\sqrt{1-x^2}\right)}{x\left(\sqrt{1+x^2}+\sqrt{1-x^2}\right)} = \lim\limits_{x\to0} \frac{2x^2}{x\left(\sqrt{1+x^2}+\sqrt{1-x^2}\right)} = \lim\limits_{x\to0} \frac{2x}{\sqrt{1+x^2}+\sqrt{1-x^2}}$
  $= 0$

- $\lim\limits_{x\to0^-} \frac{3x}{x^2+2|x|} = \lim\limits_{x\to0^-} \frac{3x}{x^2-2x} = \lim\limits_{x\to0^-} \frac{3x}{x(x-2)} = \lim\limits_{x\to0^-} \frac{3}{x-2} = -\frac{3}{2}$, car $(x<0)$

- $\lim\limits_{x\to0} \frac{2\sin x-\sin 2x}{x^2} = \lim\limits_{x\to0} \frac{2\sin x-2\sin x\cos x}{x^2} = \lim\limits_{x\to0} \frac{2\sin x(1-\cos x)}{x^2} = \lim\limits_{x\to0} 2\sin x \times \frac{1-\cos x}{x^2} = 0 \times \frac{1}{2} = 0,$
  car $(\sin 2x = 2\sin x\cos x)$